



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## IV.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF  
THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.XI. ON THE EFFECT OF TEMPERATURE ON THE VISCOSITY  
OF AIR.

BY SILAS W. HOLMAN.

Read, June 14, 1876.

THE developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.

According to the kinetic theory, the molecules of the gas are constantly in rectilinear motion. In virtue of their mass and velocity, these molecules have a certain momentum. Hence, if we have two layers of air moving over each other, we shall have a mutual interchange of momentum from the transference of molecules from one layer to another, the result being a tendency toward an equalization of the velocities of the two layers. This produces the effect of friction between the two layers, and its amount determines the viscosity of the gas in any particular case. From analytical considerations Maxwell has deduced\* an expression which, as corrected by Clausius,† should read,

$$\eta = \frac{Mu}{4\pi s^2}$$

where  $\eta$  is the coefficient of viscosity of any gas;  $M$  is the mass of a molecule;  $u$  the "velocity of mean square" of the molecules; and  $s$  the dis-

\* Phil. Mag. xix., xx.; 1860.

† Phil. Mag. xix., 434.

tance between the centres of two molecules at impact. The value of  $\eta$  is expressed in units of length, mass and time, since it is a tangential force. This formula, if true, shows that the viscosity of any gas should be independent of its density at a constant temperature, and should increase proportionally to the value of  $u$ . But  $u^2$  is proportional to the absolute temperature, whence we see that the viscosity should increase proportionally to the square root of the absolute temperature (which we may reckon from  $-273^\circ\text{C}.$ ). Maxwell has also pointed out \* that in this expression we should obtain the same result with regard to the pressure, whatever assumption we adopt of the mutual action at impact of the molecule; but that it is necessary to make some special assumption upon the nature of this action to determine the variation with the temperature.

Previous to this deduction by Maxwell, there had been but little work done upon the viscosity of gases, and almost nothing as to its variation with temperature. Subsequently, experiments have been made by Meyer, Maxwell, Puluj, and von Obermayer. The forms of apparatus used have depended upon two fundamental methods: 1°, the retardation of pendulums by the surrounding gases; 2°, the transpiration of gases through capillary tubes. In the present paper, I propose to discuss somewhat the value of these experiments in determining the variation of the viscosity with the temperature, and to describe some recent experiments made with a modification of the second of the above methods.

In a paper published in Poggendorff's *Annalen*, cxxv., 177, 1865, O. E. Meyer describes a series of experiments upon the internal friction of air made by measuring the retardation of three circular glass plates oscillating around a vertical axis in a closed receiver containing the gas, whose temperature and pressure could be varied. From the results of these measurements, Meyer concludes that the coefficient of viscosity is independent of the pressure. It will, however, be evident, upon an inspection of the published results, — especially by application of the graphical method, — that no reliance can be placed upon them for determining variation with the temperature. Meyer's second paper (Pogg. *Ann.* cxxvii., 199, 353) is devoted to a discussion of Graham's transpiration experiments,† from which we may derive quite a satisfactory proof of the law of Poiseuille as applied to gases. In the *Philosophical Transactions*, London, 1866, Maxwell published a series

---

\* *Phil. Mag.* xxxv., 211.

† *Phil. Trans. Roy. Soc. Lond.* 1846-49.

of results obtained by a similar apparatus to that used by Meyer. From these Maxwell concludes that the viscosity is independent of the pressure upon the gas, and that it increases as the first power of the absolute temperature. If, however, the results published in that paper be all upon which this law is based, we cannot regard it as very securely established. - A third paper was published by Meyer, in *Pogg. Ann.* cxliii., 14; in which the results of seven experiments with oscillating plates after Maxwell's pattern, but with bifilar suspension, were given. These, like the others, are insufficient to determine the effect of temperature. In three subsequent papers\* by Meyer a large number of experiments are described. These were made by the method of transpiration through capillary tubes, and preliminary experiments were made to prove the validity of the law of Poiseuille. This law may be expressed by the following equation: —

$$V = \frac{\pi R^4 t}{8\eta \lambda} \cdot \frac{p_1^2 - p_2^2}{2p} \quad (1.)$$

where  $V$  is the volume of gas transpired in the time  $t$ , measured at the temperature of the capillary, and under the pressure  $p$ ; the pressure at entering the tube being  $p_1$ , and at leaving it  $p_2$ . The length of the capillary is  $\lambda$ , and its radius  $R$ ;  $\eta$  being the coefficient of viscosity of the gas. This law may, I think, be regarded as established for variations of pressure not exceeding two atmospheres, and for tubes in which the length is very large as compared with the diameter.

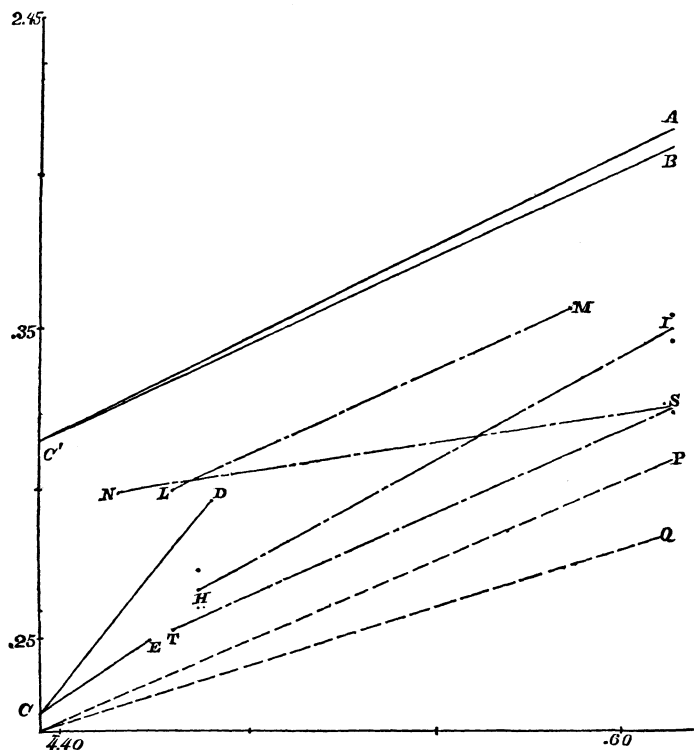
Meyer gives a series of twenty-five experiments, and selects eleven as the most reliable. These all seem to indicate an increase of viscosity with rising temperature greater than the  $\frac{1}{2}$  power, but appear at the same time quite discordant among themselves. Upon the accompanying figure, I have shown the extremes of these by a graphical representation. The method used to discuss them is one described in the *Proceedings of the Academy* for 1874, page 222. If we have a line of the general form represented by the equation  $y = mx^n$ , we may take logarithms of both sides and get the equation,  $\log y = n \log x + \log m$ , which has the form of the equation to a straight line. Hence, if we have the coördinates of a series of points which we suppose may be connected by a curve of the exponential form, we may determine this fact by plotting logarithms of these coördinates, which should give us points along a straight line whose tangent is the exponent in the primary equation. Thus, if our equation to the variation of  $\eta$  with the

---

\* *Pogg. Ann.* cxlviii., 1, 203, 526.

absolute temperature  $\tau$  be of the supposed form  $\eta = c\tau^x$ , where  $c$  is a constant, we may take the value of  $\log \eta$  and  $\log \tau$  from our experiments, and expect upon plotting them to get a straight line making an angle whose tangent is  $x$ . This method I have applied to the results of Meyer, and the extreme points are shown at the points marked  $D$  and  $E$  in the accompanying cut.

Fig. 1.



The single experiment at zero centigrade gives the point  $C$ . All the other experiments furnish points scattered between  $D$  and  $E$ . The absolute values of the coefficient in these cases are:—

	$\tau =$	$\eta =$
$C$	$273^{\circ}.C$	0.000168
$D$	$293^{\circ}.2$	0.000198
$E$	$287^{\circ}.5$	0.000178

For the line  $CD$ ,  $x=2.3$ ; for the line  $CE$ ,  $x=1.12$ . This gives us

an idea of the value of these results in determining the variation of the viscosity with the temperature. We cannot say from them, whether this variation is proportional to the first or second power of the absolute temperature. Even the results published in the fifth paper, which was to determine this law, are insufficient. In the first series of these results, shown upon the curve by the extreme lines  $NS$  and  $TS$ , we see that the exponent representing the law of variation with the temperature varies from  $x=0.21$  for line  $NS$  to  $x=0.69$  for line  $TS$ , a variation even greater than in the results previously discussed. All the other observations give points intermediate between  $N$  and  $T$ . The second series furnishes little better data; and the third series, from determinations with oscillating plates, are not sufficiently complete for discussion in this way. They, however, afford no greater satisfaction.

Puluj has used the method of transpiration for some measurements of this law, and his results appear in the Sitzber. Wien. Acad. of 1874, lxi., 287. The results which he has obtained appear rather more concordant than those of Meyer, but still show considerable disagreement. Upon the above cut, the lines  $OP$  and  $OQ$  show the extremes of these results as obtained by a discussion of his experiments. These lines do not represent the greatest variations between successive results in the same series, but the extreme variation between the mean results of various series. For  $OP$ ,  $x=0.65$ ; for  $OQ$ ,  $x=0.47$ . It will thus be seen that these results are more concordant than the different series of Meyer: they are not, however, completely satisfactory.

Later than these we have a brief notice of some experiments by von Obermayer, in the Phil. Mag., xlix., 332, 1875, in which he states that he has obtained results "which confirm those of Meyer's experiments in a perfectly satisfactory manner." He states Meyer's results as furnishing the exponent  $\frac{3}{4}$  for the variation of  $\eta$  with the absolute temperature; whence we must conclude that this number expresses the result at which he has arrived.

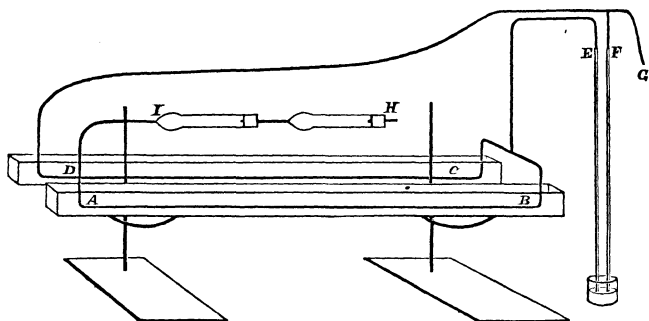
What value now are we to place upon these results, and which is the true one? Maxwell has given  $x=1$ ; Meyer,  $x=\frac{3}{4}$ ; Puluj,  $x=\frac{2}{3}$ ; von Obermayer,  $x=\frac{3}{4}$ . The first two values,  $x=1$  and  $x=\frac{3}{4}$ , we can hardly accept as certain, from the considerations previously shown. The value given by Puluj of  $x=\frac{2}{3}$  is undoubtedly somewhat greater than is warranted by his results. Of the remaining experiments we cannot judge, since they have not yet appeared in full, so far as I have been able to ascertain.

The importance of this question in its bearing upon the kinetic

theory, as well as from its prominent place among the phenomena of gases, renders it very desirable that we should know the true law.

In endeavoring to arrange some new form of apparatus for a more accurate study of this law, the idea of a differential arrangement was suggested to me by Professor Pickering. This has been the origin of the following method. Two glass capillaries, *AB* and *CD*, were placed side by side, each in a tin trough to contain a bath to regulate the temperature of the gas. Air-tight glass and rubber connectors extended from *G* to the gauge *F*, and to the end *D* of one capillary. The ends *B* and *C* of the capillaries were connected with the gauge *E* by means of a T joint of glass. The end *A* of the second tube communicated with the external air through the chloride of calcium tubes *H* and *I*. The size of the connectors at the ends of *AB*

Fig. 2.



and *CD* was sufficient to allow the gas to assume the temperature of the bath. The tube at *G* was connected with a large flask, from which the air was continuously exhausted by means of a Richards' jet aspirator. The size of this flask rendered the pressure constant in spite of slight variations in water pressure. An inspection of this arrangement will show that when the flask is exhausted, and a vacuum produced at *G*, the air will enter at *A* under the atmospheric pressure, and will pass with constantly diminishing pressure to *G*; so that, at any intermediate point, as the junction of the two tubes at *BC*, we shall have a pressure intermediate between the two extremes. It will also be seen that the same volume of air is successively transpired through *AB* and *CD*; providing that there be no leak, which was carefully guarded against by making all the joints about *C*, *B*, and *E*—which were the only ones that affected the results—as tight as possible. By the two baths we may have the gas transpired successively through *AB* and

$CD$ , either at the same or at different temperatures. Now, if we denote by  $V_1$ ,  $R_1$ ,  $\lambda_1$ ,  $\eta_1$ , &c., the volume of gas transpired by  $AB$ , the radius and length of  $AB$ , and the coefficient of viscosity of the air passing through it, while  $V_2$ , &c., represent the same quantities for  $CD$ ; also, if  $p_1$ ,  $p_2$ ,  $p_3$  represent the pressure of the gas at  $A$ ,  $B$ ,  $C$ , and  $D$  respectively as obtained from the gauge and barometer readings; then from (1) we may write,

$$V_1 = \frac{\pi R_1^4 t_1}{8 \eta_1 \lambda_1} \cdot \frac{p_1^2 - p_2^2}{2\rho} \quad (2.)$$

and

$$V_2 = \frac{\pi R_2^4 t_2}{8 \eta_2 \lambda_2} \cdot \frac{p_2^2 - p_3^2}{2\rho} \quad (3.)$$

But if both baths are at the same temperature  $V_1 = V_2$  if  $t_1 = t_2$ , and  $\eta_1 = \eta_2$ , whence we may write,

$$\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1} = \frac{p_2^2 - p_3^2}{p_1^2 - p_2^2} \quad (4.)$$

Also in general it will be seen from the nature of the apparatus that  $\frac{V_1}{1+a\delta_1} = \frac{V_2}{1+a\delta_2}$ , where  $\delta_1$  and  $\delta_2$  represent respectively the temperatures at which  $V_1$  and  $V_2$  are transpired. Hence

$$\frac{\eta_1}{\eta_2} = \frac{R_1^4 \lambda_2}{R_2^4 \lambda_1} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \quad (5.)$$

From equation (5) it will be seen that, in order to determine with this apparatus the ratio  $\eta_1 : \eta_2$ , between the coefficients of viscosity in the two tubes when the temperature of these is  $\delta_1$  and  $\delta_2$  respectively, we have only to know the ratio of the dimensions as expressed by  $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$ , and to measure  $p_1$ ,  $p_2$ , and  $p_3$  by reading three mercury columns. Also we can obtain a value of  $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$  from readings of the gauges when  $\delta_1 = \delta_2$ , which needs only to be corrected for expansion of the glass to be used directly in equation (5). The whole process is thus reduced to the simple matter of reading columns of mercury, no measurements of volumes of gas being necessary. The nature of the correction of  $R$  and  $\lambda$  for temperature appears by putting into the above formulæ in which these values are supposed to be for  $0^\circ \text{C}$ , the coefficients of expansion of the glass  $= A$ ; we thus get from (5) :—

$$\begin{aligned} \frac{\eta_1}{\eta_2} &= \frac{R_1^4 (1+A\delta_1)^4 \lambda_2 (1+A\delta_2)}{R_2^4 (1+A\delta_2)^4 \lambda_1 (1+A\delta_1)} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \\ &= \frac{R_1^4 (1+A\delta_1)^3 \lambda_2}{R_2^4 (1+A\delta_2)^3 \lambda_1} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \end{aligned} \quad (6.)$$



Lest, however, an error might occur in the last reduction from a difference between the coefficient of expansion of the bore of a capillary tube and of its lineal expansion, I have carefully measured both, and find that the coefficient for the bore is 0.0000075, while for the linear expansion I find 0.0000080 per degree centigrade, a difference too slight to affect the results in my use of it; I have thought it best to use the value 0.0000075 as it entered in the fourth power, while the other entered only in the first power. The tubes used have also been calibrated to insure the selection of those of uniform bore, and their dimensions have been accurately measured by mercury and a micrometer screw. The dimensions of the two tubes used in the experiments to be described, were, for tube No. I.,  $\lambda = 1272.3$  mm.,  $R = 0.1098$  mm.; for tube No. II.,  $\lambda = 1274.1$  mm.,  $R = 0.1115$  mm.

To make an experiment with this apparatus, it is merely necessary to start the jet of water and allow the exhaustion to proceed until the mercury columns in *F* and *E* have come completely to rest. Readings are then taken of the heights of these columns by means of a cathetometer from a steel scale placed beside the gauges. The reading of the barometer corrected for instrumental error gives the pressure at *A*. All these are reduced to the freezing point, and *E* and *F* are corrected for capillarity by the tables of Delcros. The temperature of the baths is also taken by thermometers in various positions in the troughs. This must be kept constant throughout the experiment, and I have, therefore, principally used the temperatures of melting ice and boiling water. In the experiments of which the following table gives the results, advantage has been taken of the four methods of checking the results of one experiment by another, by reversing the direction of flow of the air through the tubes and heating alternately, in each case, first one and then the other trough. In the table, column first gives the number of the experiment; column second, the direction of flow of the air, which entered at the tube whose number is first given and passed out from the other; columns three, four and five give the pressures at *A*, *B* and *D* respectively; columns six and seven show the temperatures in centigrade degrees of the baths around tubes I. and II. respectively; column eight shows the values of the ratio  $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$  at different tempera-

tures; column nine, the values of  $\frac{\eta_1}{\eta_2}$ , i.e. of  $\eta$  at the higher to  $\eta$  at the lower temperature; column ten shows the values of the exponent  $\alpha$  in the equation  $\eta = c\tau^\alpha$ . This is the quantity which it was the object of the experiments to obtain.

No.	Dir.	$p_1$	$p_2$	$p_3$	$T_L$	$T_{IL}$	$\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$	$\frac{\eta_1}{\eta_2}$	$x$
1	I.-II.	<i>m. m.</i> 759.9	<i>m. m.</i> 525.2	<i>m. m.</i> 16.3	17.0	17.0	0.912	1.083	0.799
2	"	"	549.3	17.1	17.0	47.5			
4	"	759.8	525.6	18.0	15.1	15.1	0.916		
5	"	"	584.4	18.9	"	"	0.921		
6	"	765.7	550.9	18.6	17.8	17.8	0.934		
7	II.-I.	"	490.7	17.7	17.5	99.0		1.212	0.776
8	"	"	491.2	17.6	17.5	99.5		1.206	0.755
9	"	"	490.0	17.3	17.5	99.8		1.215	0.780
11	"	755.2	467.8	20.4	0.0	100.0		1.272	0.771
12	"	"	468.4	19.4	"	"		1.267	0.757
13	"	"	467.9	19.6	"	"		1.271	0.768
14	"	"	467.7	19.3	"	"		1.273	0.773
16	"	"	544.2	20.7	0.0	0.0	0.927		
17	I.-II.	756.7	525.3	23.4	"	"	0.928		
18	"	"	594.8	21.5	0.0	100.0		1.277	0.782
19	"	761.4	529.1	16.1	100.0	100.0	0.933		
20	"	762.0	530.2	16.7	"	"	0.937		
21	"	763.1	452.2	18.5	100.0	0.0		1.259	0.738

In the calculation of the ratio  $\frac{\eta_1}{\eta_2}$  of this table, the value of  $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$  used was the mean of that obtained from experiments 16 and 17, after correcting for temperature. The agreement of these two values within 0.1 per cent is a test of the accuracy of the method, as the two experiments were made on different days, and the direction of the current was reversed. It will be seen that the value of this quantity increases slightly with the temperature, as we should expect from the slight difference in size of the two tubes used. The values of  $x$  will be seen to agree quite closely, with the exception of experiments 2 and 21. I have treated these results in the same manner as those of Meyer, and the result is shown on Fig. 1.

The point  $A$  is plotted from experiment 18, and  $B$  from 21; so that the lines  $AC'$  and  $BC'$  show the greatest variation in nine out of ten determinations, while the majority of these lie so close together as not to be capable of clear representation between  $A$  and  $B$ . The point  $C'$  has been raised from  $C$  for distinctness. Experiment 2 would indicate a deviation from the straight line; but I do not regard this as a perfectly reliable determination. More experiments are needed between  $0^\circ$  and  $100^\circ$  to establish the law.

In order to compare these results with those of Meyer, I have been obliged to assume his value of  $\eta = 0.000168$  at  $0^\circ \text{C.}$  as a starting-point, since the apparatus which I have used does not give absolute values of the coefficient of viscosity, but only ratios. It would appear, however, that the great concordance among the results thus far obtained would warrant its application to absolute measurements, for

which it would only be necessary to measure the volume of the gas transpired in a known time. These, with experiments upon other gases, and also upon the validity of Poiseuille's law, I hope to be able to accomplish. The many points of superiority of this apparatus, and the excellence of these preliminary results, would seem to indicate more accurate determinations than others preceding them.

As a result of these experiments, it would appear that the viscosity of air increases proportionally to the 0.77 power, nearly, of the absolute temperature between  $0^{\circ}$  and  $100^{\circ}$  C. This value corresponds quite closely to the  $\frac{3}{4}$  power, and we might infer that this was the value of  $x$  towards which the experiments pointed; but as I feel assured that further experiments will furnish still more concordant results, I should be unwilling to accept 0.75 until these had been performed. The general agreement of my results with the numbers of Meyer and von Obermayer would seem to point to the fact that the value of  $x$  cannot be as great as unity, and is probably about 0.75.